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VIII.

ON PORTABLE ASTRONOMICAL INSTRUMENTS AND THEIR USE.

By Truman Henry Safford.

Presented, Oct. 12, 1875.

In determining time, latitude, and the azimuth of a meridian-mark, which are the principal operations of geodetic astronomy, the only instrument now much used in America (besides the sextant) is the portable transit, with added apparatus for speedy reversal, and a micrometer for differences of zenith distance, according to Talcott's method. A portable transit so constructed affords a speedy determination of all three elements mentioned above; but its practical handling is a little difficult to an astronomer used only to large fixed instruments, and the determination is not always the most economical in time and labor of observation and reduction.

The present paper is intended to give a few practical hints derived from actual experience with portable instruments of all grades, from an ancient transit by a forgotten maker, with an object-glass which would not come to a focus, up to the latest productions of the best workshops of America, England, and Russia.

The beginner will do well to practise with an instrument which is not quite perfect: he thus learns in an exaggerated form all the faults to which instruments are liable.

I need not describe these instruments, but will simply refer to Chauvenet's admirable Manual of Practical and Spherical Astronomy, also to the Report of the Coast Survey for 1866. I will suppose a latitude-and-longitude campaign to be planned. The first matter to be settled is, what instruments are to be used? If the work be simply geographical, without special requirements of extreme precision, like the boundaries established by the United-States Land Office, and especially if the country be rough and transportation very difficult, a small transit instrument will suffice. I am inclined to think a focal length of 14 to 16 inches, and aperture of the object-glass of 1¾ inches, are the best dimensions. The frame should be rather light, but solidly put together; the setting circle plainly divided; the striding level delicate (2″ to one division); and, above all, the apparatus for illuminating the field as perfect as can be made.

Such an instrument ought to show a star of the sixth magnitude with full illumination, if the observer's eye be accustomed to not too bright a light, and especially if he use Mr. Rogers's rulings on glass in place of spider-lines: indeed, he will hardly fail to do so, for other reasons. The instrument of this size with which I am acquainted, belonging to the Canadian government, was planned as an alt-azimuth by Lindsay Russell, Esq., Deputy Surveyor-General, and made by Simms of London. The star λ Ursæ minoris of the 6.7 magnitude could be readily observed with it. It is not too heavy, with all the attachments, to be carried on a strong man's back; nor too large to accompany the observer in a sleeping-car. A somewhat larger transit, by Temple of Boston, did excellent service on the south boundary of Wyoming. This has a two-inch aperture, a pretty long focal distance, but a short axis and a light frame. It looks ill-proportioned, owing to the length of its telescope; but has a very excellent object-glass. Its greatest fault is instability in collimation; the telescope tube seems weakly put together; and the mounting, as I used it, was unstable too, probably because it was fastened to a plank on a wooden post.

To mount such an instrument away from civilization requires a good deal of trouble and expense. Brick is, of course, the best material for the foundation, but cannot always be obtained; and, at one of my stations, the only two brick-masons in town were intoxicated, and the pier was built by a civil engineer who accompanied me, with a sergeant of the United-States Engineers to mix the mortar. At Duluth the ground itself furnished rough stone in place.

At Santa Fé an unfinished and abandoned state-house furnished a pier of cut stone. At Fort Union, the sun-dial of the fort, removing the gnomon, was an excellent pillar for the instrument. Chauvenet's suggestion to use a tree-stump is impractical, on account of the roots: the instrument is kept in constant tremor by persons walking about.

When circumstances compel the use of a wooden post, great care must be taken to shield it from the sun, and the observations must be so distributed that the changes in azimuth and level are harmless. The level requires constant watching, but ought not to be changed during a group of stars, lest the azimuth be disturbed too. To eliminate changes in the latter, the groups of four or five stars must take but little time.

All these things considered, it is best to use the ephemerides of 529 stars yearly published at Berlin, originally intended for the reduction of the great zones now in progress. I think it altogether likely that this will be eventually the standard time-list for astronomers generally in this hemisphere : at any rate, these places will be kept accurate by continual observation for a good many years to come. They include all stars north of 10° of *south* declination down to the fourth magnitude, and a selected list of fainter ones to fill gaps. Many of these same stars are also given in the Connaissance des Temps, the Nautical Almanac, and the American Ephemeris ; but these latter have not quite enough of them for rapid work.

To illustrate how azimuth and collimation are to be determined and eliminated, I give a scheme of observations actually used at Pueblo, C. T., May 11, 1873, by Lieut. E. H. Ruffner of the United-States Engineers. The scheme was, in general, agreed upon between him and myself.

Star.	Position of instrument.	A. R.	Decl.
		h. m. s.	
γ Ursæ majoris .	I.	11 47 10	+54°24′
ο Virginis . . .	I.	58 45	9 26
4 H. Draconis . .	I.	12 6 19	78 20
η Virginis . . .	I.	13 26	0 2
6 Canum . . .	II.	19 37	39 43
20 Comæ . . .	II.	23 22	21 36
κ Draconis . . .	II.	28 7	+70 29
γ Virginis . . .	II.	35 15	— 0 45
ε Ursæ majoris .	II.	48 29	+56 39

On comparing this with other similar schemes, it will be noticed that here are no stars observed below the pole, and but one south of the equator; in other words, all are as near the zenith as practicable. Moreover, every pair of consecutive stars include the zenith between them (this is not absolutely essential, but yet very well), and hence give a definite value of the clock correction if the collimation be known.

My own scheme corresponding to the above was as follows : —

Denver, May 11.

Star's name.	Position of instrument.	A. R.	Decl.
		h. m. s.	
ξ Ursæ majoris . .	I.	11 11 25	32°15′
ι Leonis	I.	17 19	11 14
λ Draconis . . .	I.	23 53	70 2
υ Leonis	I.	30 28	— 0 7
3 Draconis . . .	I.	35 25	67 27
χ Ursæ majoris .	I.	39 22	48 29
γ Ursæ majoris .	II.	47 10	54 24
ο Virginis . . .	II.	58 45	9 26
4 H. Draconis . .	II.	12 6 19	78 20
2 Canum . . .	II.	9 47	41 22
η Virginis . . .	II.	13 26	0 2

In determining latitude with an instrument of small size, Talcott's method is subject to some inconveniences, which may sometimes be better avoided by using Bessel's method; that is, by establishing the transit in the prime vertical.

But here the same principle (immediate elimination of azimuth error) ought to be carried out; that is, the same stars should not, in general, be used east and west of the meridian. This process is Bessel's own, as distinguished from Struve's.

I annex a scheme of observation (from the Report for 1873–74

Star's name.	E. or W. transit.	Position of instrument.	Time of transit.	A. R.	Decl.
			h. m.	h. m. s.	
ρ Herculis . . .	W.	I.	20 17	17 19 19	37°16′
6 Lacertæ . . .	E.	I.	22	22 25 3	42 29
ο Andromedæ . .	E.	I.	43	22 56 7	41 39
θ Herculis . . .	W.	I.	50	17 51 54	37 16
ζ Andromedæ . .	E.	II.	21 19	0 10 30	37 59
ι Andromedæ . .	E.	II.	30	23 31 57	42 34
ζ Lyræ	W.	II.	36	18 40 25	37 29
η Lyræ	W.	II.	52	19 9 27	38 56
μ Andromedæ . .	E.	II.	56	0 49 45	37 49
θ Lyræ	W.	II.	22 4	19 11 59	37 55
γ Cygni	W.	II.	51	20 17 42	39 51
τ Andromedæ . .	E.	II.	23 1	1 33 9	39 56
υ Andromedæ . .	E.	II.	6	1 29 24	40 46
ν Cygni	W.	II.	17	20 52 28	40 41
75 Cygni . . .	W.	I.	35	21 35 14	42 42
γ Andromedæ . .	E.	I.	44	1 56 10	41 43
16 Persei . . .	E.	I.	50	2 42 38	37 48
σ Cygni . . .	W.	I.	56	21 12 47	38 52

of the Chief Engineer of General Sheridan's staff) which I used for this purpose at Bismarck, D. T., Oct. 9, 1873. It was my first fair trial of the method. The stars were selected from my catalogue of 981 stars, published in that year by the War Department, and are all thoroughly well determined, — an advantage which Bessel's method has had over Talcott's.

This scheme requires four hours observing. If I recollect rightly, it was somewhat interfered with by clouds, and would otherwise have been sooner finished. I was anxious to succeed that evening, as time pressed; and I therefore did not attempt to accumulate as many observations as possible, but preferred to make sure of a few. The result was sufficiently accurate for geographical purposes, having a probable error of ± 0″31. The instrument was quite indifferent in its optical portion. The Land Office at that time required a probable error of less than 3″; and does still, so far as I know. There are Land Office determinations extant which are far more than this in error.

In more precise latitude and longitude work, the instruments used have generally 3 inches aperture, and 30 to 36 inches focal length. Such an instrument should be very solidly built and set up. The one with which I am most familiar is Brigham Young's, in the Temple yard at Salt Lake City: it is by Würdemann, and was originally placed there at the time of the Coast Survey determination of longitude at that place. I found it very firm and strong: its level, collimation, and azimuth errors were constant, though not very small, as the Mormon astronomers seem not quite expert in adjusting. But I did not for this reason neglect to make the full number of observations, nor to distribute them precisely as if I were observing with a smaller and worse instrument, partly from habit, and partly because the instrument at Evanston, W. T., with which I was comparing time, was the instrument by J. H. Temple, mentioned above; and the observers were exchanged in the middle of the series.

There are two essentially different patterns of large portable transits. The one, the German or Russian, has a prism between the object-glass and the eye-piece; and the eye-piece itself is at one end of the horizontal axis. I have used such instruments only for trials of personal equation. There is one such at the Harvard College Observatory, and others were made for the American Transit of Venus expeditions. These transits are very convenient. The level is always upon the axis. The observer sits in one position between reversals, and has not the troublesome necessity of bending his body into inconvenient postures. His setting circle is directly before him, his working list at one side, and his chronometer or telegraph-key at the other. But he is liable to a troublesome personal equation; and I am told that the collimation cannot easily be made steady, owing to the great prism between objective and ocular. The instruments are also very costly; so that, in this country, a construction is preferred which is nearer like the ordinary observatory transit. Here the prismatic or diagonal eye-piece takes the place of the "broken telescope," as the other construction is technically called in German. The observer has to change position, and is liable to a variety of petty annoyances thence arising. Upon the whole, I think one construction is as good as the other for practical purposes.

The distribution of stars to be observed for time may often be improved by employing one star within 10° of the pole to every group of four or five time-stars. The latter will then be predominately south of the zenith, but not exclusively so. Where the instrument is known to be very firm and solidly mounted, and has a reversing apparatus, the collimation may be determined by the pole-stars alone. The double-group for Denver, as previously given, would be modified by introducing the polars 39 Cephei Hevelii and 6 Ursæ minoris Bode, as follows: —

Star's name.	A. R.	Decl.
ξ Ursæ majoris ι Leonis } as before.	h. m. s.	
58 Ursæ majoris	11 23 38	+43°52′
39 Cephei H. sub-polo . .	23 27 47	86 37
χ Ursæ majoris	11 39 22	48 29

INSTRUMENT REVERSED.

Star's name.	A. R.	Decl.
	h. m. s.	
β Virginis	11 44 5	2 29
γ Ursæ majoris o Virginis 2 Canum } as before		
6 Ursæ minoris B.	12 14 23	88 24

The single polar in the first position (39 Cephei H.) is probably quite sufficient to give equal accuracy in azimuth with the two (λ and 3 Draconis) in the former list, especially as 4 Draconis H. is now replaced by a close polar. The intervals are now a little closer in some cases than before, as the instrument is expected to be easier reversed by a machine, and the lines in the focus to be nearer together.

Additional time-stars may also be inserted, namely: —

Star's name.	A. R.	Decl.
	h. m. s.	
τ Leonis	11 21 23	3°33′
ω Virginis	11 31 53	8 50
π Virginis	11 54 33	7 19
4 Comæ Ber.	12 5 24	26 35

But this would involve (as indeed the close polars do) much extra computation.

The fashion in Germany has been, lately, to select a list of stars to be observed regularly, night after night, at both stations; thus freeing the results more exactly from errors in the star-places, and, indeed, supplying observed right ascensions of great accuracy. This plan would be excellently well adapted for such observations as those between Denver and Pueblo, which are not far apart in longitude; but in much American work the distances are too great, and the use of the telegraph-lines too precarious. Good star-places are common enough, if we have them all collected in a convenient place; and this is done in the German catalogue mentioned, and, when that has not enough, in my own catalogue of 981 stars, which is soon to be doubled in extent, so as to include the zone between 10° and 70° of declination, instead of 30° to 60° only.

The young observer should by no means fail to accustom himself to the eye-and-ear method of observing. It is, as Leverrier has remarked, a better discipline than the chronographic. It is somewhat less accurate: but one who can use it skilfully can always adapt himself to a chronograph with ease; and, on the other hand, if the chronograph breaks down, or any trouble with it occurs where help cannot be got, the eye-and-ear observer is more independent than the mere chronographist. For this reason, I have never allowed a pupil to use a chronograph till he had mastered the elementary practice of the other method. In my own experience at Chicago upon the great zones I had no chronograph, but did not find that a serious drawback. Argelander was on this point ultra-conservative; but I do not think there are many observers by the new method whose work I would take in exchange for his by the older. The temptation with mechanical methods of observing is to undertake more work than can be reduced; which is a bad practice.

In determination of time, Döllen has suggested using the transit in the vertical of Polaris. I have often used this method on a first night's work, but, when the chronometer error is roughly known, can never resist the temptation to bring the instrument at once pretty close to the meridian. The pupil should be thoroughly practised in the minutiæ of this process; for I have seen even a good observer badly vexed with it, when an instrument new to him, especially a poor one, was employed.

In the determination of latitude by Talcott's method, I think a device of Mr. Rogers is likely to be useful. He replaces the micrometer by a system of parallel lines oblique to the meridian, so that each star must pass two sets of three each in its transit. The lines are very beautifully and exactly ruled by his process, and they save the time required to turn the micrometer screw. If this has to be moved largely, a star might often be lost; which is the more troublesome, as in doubtful weather, when there are flying clouds, the pairs are often spoilt by losing one of the stars. It is quite probable that Mr. Rogers's improvement will enable us to go farther from the centre of the field, and thus help the choice of star-places.

The stars should always be so chosen that the positive and negative distances from the centre of the field $[\frac{1}{2}(\delta + \delta') - \varphi]$ may pretty nearly balance, so as to give the means of determining the micrometer-values from the latitude observations themselves; otherwise the resulting latitudes could not be as accurate as the star-places and observations would permit.

The foreign astronomers use Talcott's method very little. Their objection to it is, that the star-places must necessarily be worse than for Bessel's or Struve's, or for the employment of a vertical circle or portable meridian circle. These latter instruments, however, are not used in America, principally because our methods were formed independently of the modern Germans, partly because we have no very good dividing-engines, and because they are too delicate to stand transportation over our frightful Western roads. On the other hand, Talcott's method has greatly helped our astronomers by furnishing a definite aim for their meridian observations and compilations of star-catalogues, and will thus contribute largely to the knowledge of stellar proper motions, as I shall show elsewhere. I may here say, that the probable error of a star's declination, as compiled from the best authorities, can be so estimated, classified according to the quantity and goodness of the materials, and allowing enough to cover all defects.

Class of Star.	Probable Error of Declination.
AA.	0″18
A.	0 28
B.	0 43
C.	0 70

Within 30° of the zenith, there are now stars enough of the classes AA, A, and B, for any American latitude, using Talcott's method; and the probable error of declination of a single pair of stars will vary from 0″.13 to 0″.31. Allowing, then, a p. e. of 0″.43 to each observation, we have the final probable error of latitude from a pair of stars, observed thrice, —

$$= \sqrt{0''.13^2 + \frac{0.43^2}{3}} \text{ or } \sqrt{0.31^2 + \frac{0.43^2}{3}};$$

that is, from ± 0″.28 to ± 0″.40; which only requires from eight to sixteen (say twelve) pairs to give a probable error of 0″.1.

Moreover, observations now in progress, both general and special, will in a year or two raise all the stars of the British Association's catalogue now classed as C (within our latitude-limits) to the class B, and will doubtless transfer many of this class to a higher. On the other hand, the portable meridian circle, or the prime vertical transit, needs only stars of the two highest classes, and, if other practical difficulties do not intervene, can probably secure this same degree of accuracy with fewer observations; not necessarily, however, with a less amount of time and trouble. I am inclined to think that the extravagant praises of Talcott's method to which our officials give utterance are about balanced by the steady adherence of the French and Germans to the other way, and that the practical difference between them is one rather of habit than essential. It is quite certain that both give excellent results.

The reduction of the observations for time and latitude is simple enough, and the methods are given in the ordinary books. Some discretion, however, is desirable in applying them.

The application of least squares to time reductions is considered by Struve often unnecessary; nor is it generally practised in Germany and Russia. Where it is applied, weights should be given to the observations depending upon the star's declinations. I am inclined, in case the observations are fairly complete, and depend on about the same number of wires, to consider the expression

$$\varepsilon^{\circ}\sqrt{1+\sec\delta^{2}}$$

as a fair representation of the probable error in different declinations. Hence the weight will be expressed by

$$\omega=\frac{2}{1+\sec\delta^{2}},$$

that at the equator being taken as unity. If azimuth, collimation, and clock-error, or rather small corrections of their adopted values, are the unknown quantities, their co-efficients, multiplied by $\sqrt{\omega}$, will be

$$\begin{aligned}
A\sqrt{\omega}&=\sin(\varphi-\delta)\sec\delta\sqrt{\frac{2}{1+\sec\delta^{2}}}=\sin(\varphi-\delta)\sqrt{\frac{2}{1+\cos\delta^{2}}}\\
C\sqrt{\omega}&=\sqrt{\frac{2}{1+\cos\delta^{2}}}\\
\sqrt{\omega}&=\sqrt{\frac{2}{1+\sec\delta^{2}}}
\end{aligned}$$

and their required squares and products

$$\begin{aligned}
A^{2}\,\omega&=\sin^{2}(\varphi-\delta)\,C^{2}\omega\\
AC\omega&=\sin\,(\varphi-\delta)\,C^{2}\omega\\
C^{2}\omega&=\frac{2}{1+\cos\delta^{2}}\\
A\omega&=\sin(\varphi-\delta)\,C\omega\\
C\omega&=\frac{2}{\sec\delta+\cos\delta}\\
\omega&=\frac{2}{1+\sec\delta^{2}}.
\end{aligned}$$

I have tabulated the values of $C^{2}\omega$, $C\omega$, and ω, together with their logarithms, according to these formulæ, and give them in Table I. For any station, the preparation of $A^{2}\omega$, $AC\omega$, $A\omega$, is at once very simple.

The best results are not obtained from poor observations by *cooking* them, but by letting them pass for what they are worth. Of course, an observation must now and then be rejected. Argelander's method was to scrutinize doubtful cases with much care, adopting or rejecting them, and giving the results of both methods. His experience in his own line was so great, that he rarely missed assigning the probable cause for any large discrepancy, whether it arose from errors in reading off mistakes in wires, miscounting time, or imperfect hearing by the recorder, where one was employed, as in the Histoire Céleste; and, if he was at fault, he would suspend judgment on the case, and note it down for further observation. In geographical work, where the observer must finish each problem in a given time, and is comparatively thrown upon his own resources for little repairs to his instruments, and the means of avoiding their occasional great defects, natural or acquired, he must proceed with double caution in making his observations, checking them in every way, and making enough to get his result in spite of any abnormal discrepancies. This is the great advantage of simplicity in the field-work, and reliance upon the star-catalogues (which can always be improved afterwards), for what they will give.

The reduction of latitude observations by the zenith telescope needs but little remark. The process is a simple one: the usual form (see United-States Coast Survey Report for 1866) unnecessarily complicated.

The half-sum of declinations of the two stars can be at once computed; first, the mean value for the beginning of the year, $\frac{1}{2}(\delta_1 + \delta_2)$, and then the half-sum of apparent declinations for the date: thus from

$$\delta'_1 = \delta_1 + Aa'_1 + Bb'_1 + Cc'_1 + Dd'_1 + \tau\mu'_1$$
$$\delta'_2 = \delta_2 + Aa'_2 + Bb'_2 + Cc'_2 + Dd'_2 + \tau\mu'_2$$

it follows that

$$\begin{aligned}\tfrac{1}{2}(\delta'_1 + \delta'_2) = \tfrac{1}{2}&(\delta_1 + \delta_2)\\ &+ A\,\frac{a'_1 + a'_2}{2}\\ &+ B\,\frac{b'_1 + b'_2}{2}\\ &+ C\,\frac{c'_1 + c'_2}{2}\\ &+ D\,\frac{d'_1 + d'_2}{2}\\ &+ \tau\,\frac{\mu'_1 + \mu'_2}{2}\end{aligned}$$

That a'_1, a'_2, and the rest, are only given by their *logarithms*, is no objection to the use of this formula. The computer has simply to employ the Gaussian logarithms. The best four-place table I know (J. H. Traugott Müller's, 2d edition, Halle, 1860) has them in excellent shape for this purpose. The micrometrical and refraction corrections should be placed in one column, and computed together by a small table of the value of one division or its logarithm, as affected by refraction at various altitudes. A very trifling correction from the usual table is necessary in Rocky Mountain work, as the barometer may stand at 23 or 24 inches instead of 30.

The form of reduction which I suggest will be found in Lieut. Wheeler's report on the geographical positions of Cheyenne and Colorado Springs.

In my catalogue of 981 stars, the logarithms of a' b' c' d' are given for 1875, and will serve for some years to come. The trifle of error introduced by their use after the lapse of a few years can best be corrected by selecting some few stars, and computing their reductions to mean place, say for 1975, thus getting the correction for 100 years; or by differential formulæ. These will be

$$\frac{da'}{dt} = \frac{\text{Secular variation}}{100}$$

$$\frac{db'}{dt} = -\cos\alpha \frac{d\alpha}{dt} = -aa'\frac{15}{n}\sin 1''$$

$$\frac{dc'}{dt} = [-\tan\omega\sin\delta - \sin\alpha\cos\delta]\frac{d\delta}{dt} - \cos\alpha\sin\delta\frac{da}{dt}$$

$$= -[15\,ad' + a'c'\tan\delta + 15\,a'd]\sin 1''$$

$$\frac{dd'}{dt} = -\sin\alpha\sin\delta\frac{da}{dt} + \cos\alpha\cos\delta\frac{d\delta}{dt} = [15\,ab'\sin\delta$$

$$+\frac{a'^2}{n}\cos\delta]\sin 1''$$

In computing the probable error of the latitude determinations, I should proceed as follows: —

The stars should be classified, and the probable error of the catalogue declination of each class estimated, as suggested above. The comparison of observations of the same pair will give the probable error of observation. The mean *reciprocal* of the number of observations on each pair should be taken: its reciprocal will give the average weight of a pair as depending on this circumstance only.

The pairs should now be classified by computing the probable error to be expected, owing to both causes: those pairs which are once or twice observed, or whose stars are both doubtful (Class C), will give a large *à priori* probable error. These probable errors should now be compared with the actual ones, to ascertain if any error constant to each pair exists ; which ought not to be, but is often. The weights to each pair being now roughly assigned, the observations should be treated by least squares (if they can be improved in this way), considering the latitude and the value of one micrometer revolution as the unknown quantities.

A weak point in the zenith telescope is the connection of the level readings with the actual position of the vertical axis; arising from the fact that the level has to be much handled, and to be tilted in observing. It might be well, therefore, to employ the delicate level only for the reading off, and have a separate rougher one for the setting circle; placing the former in direct connection with the vertical axis. The instrument ought to be so constructed, that the two delicate levels used for time and latitude respectively could replace one another ; saving one spare level, or else diminishing the chance of loss from their breakage. Some of the earliest as well as of the latest meridian and equal altitude instruments are reversed by a machine, instead of being turned around a vertical axis. I think this is an improvement in solidity, if not in rapidity of observing.

I will give, as an example of a method of discussing latitude observations, the latitude of Colorado Springs as observed by Dr. Kampf. (See Lieut. Wheeler's Report on Cheyenne and Colorado Springs, pp. 70ff.) The stars are taken either from my catalogue of 981 stars, or computed by myself on similar principles: the quantities $\Delta\varphi$ are here added from a completer discussion of the declinations than given in the Report.

PAIRS ONCE OBSERVED.

Pairs No.	$\Delta\ \phi$	ϕ	Class of stars.	Probable error. (1)	Probable error. (2)
1	+0″76	38°49′43″21	BA	±0″49	
4		41. 22	AB	±0. 49	
8	+0. 90	40. 59	AB	±0. 49	
9	—0. 20	41. 22	BA	±0. 49	
10	—0. 20	40. 84	AB	±0. 49	
11	+0. 25	40. 05	AA	±0. 46	
12		40. 08	CA	±0. 57	
13	—1. 68	41. 23	AB	±0. 49	
	Mean	41. 06	Mean	±0″50	±0″60

All these observations except those of pair 4 were taken on one day, August 2d, on which a constant difference of about — 0″.7 from the final result is exhibited, save for the first pair. The probable errors (1) are derived from the estimated values as derived from the separate stars, and the probable error of one observation ±0″428 as given by Dr. Kampf.

PAIRS TWICE OBSERVED.

Pairs No.	Δ φ	φ	Class of stars.	Probable error. (1)	Probable error. (2)
2	— 0″76	38°49′41″60	AC	0″48	
3	— 0. 15	41. 29	AA	0. 35	
5	+ 1. 18	41. 38	AB	0. 39	
6	+ 0. 24	41. 04	AA	0. 35	
7	+ 0. 09	40. 90	AA	0. 35	
14	+ 0. 25	41. 00	BC	0. 45	
15		41. 36	AC	0. 48	
16	— 0. 05	43. 04	CA	0. 48	
17	— 0. 40	41. 41	AA	0. 35	
18	+ 0. 25	41. 56	CB	0. 45	
19	+ 0. 43	41. 75	CA	0. 48	
20		41. 42	BA	0. 39	
21		41. 25	AA	0. 35	
22	— 0. 10	41. 72	AC	0. 48	
23	+ 0. 45	41. 67	AC	0. 48	
24	— 0. 24	41. 17	BA	0. 39	
49	— 0. 10	41. 92	AB	0. 39	
50	0. 00	42. 52	AA	0. 35	
		Mean 41. 56	Mean	±0. 42	±0″323

PAIRS THREE TO SIX TIMES OBSERVED.

AVERAGE NUMBER, FOUR.*

Pairs No.	Δ φ	φ	Class of stars.	Probable error. (1)	Probable error. (2)
25	— 0″15	38°49′42″04	AA	±0″27	
26	+ 0. 04	40. 91	BB	±0. 37	
27	+ 0. 90	41. 78	AC	±0. 43	
28	— 0. 35	41. 87	AA	±0. 27	
29		41. 21	BA	±0. 33	
30	— 0. 45	40. 35	CA	±0. 43	
31	+ 0. 42	42. 38	BA	±0. 33	
32	+ 0. 10	42. 36	BC	±0. 46	
33		41. 52	BB	±0. 37	
34		40. 96	BB	±0. 37	
35		42. 40	AC	±0. 43	

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* The number of observations now makes but a trifling difference in the probable errors.

Pairs No.	Δ φ	φ	Class of stars.	Probable error. (1)	Probable error. (2)
36		38°59′42″00	BB	±0″37	
37	—0″19	42. 62	AA	±0. 27	
38	—0. 14	42. 31	AA	±0. 27	
39	+0. 08	41. 84	AB	±0. 33	
40	+0. 08	41. 06	AC	±0. 43	
41	—0. 46	41. 60	AA	±0. 27	
42	—0. 46	41. 27	AA	±0. 27	
43	—0. 29	41. 35	BA	±0. 33	
44	0.	42. 69	AA	±0. 27	
45	0.	42. 46	BC	±0. 46	
46	—0. 38	40. 78	BA	±0. 33	
47	—0. 91	40. 92	BA	±0. 33	
48	—0. 91	40. 12	CA	±0. 43	
51	+0. 38	40. 99	AA	±0. 27	
		38 59 41 63		±0. 357	±0. 527

Classifying according to magnitude of probable error (1), —

Class (α). All stars AA more than once observed.
All stars AB more than twice observed.

20 pairs of class (α) . . . . 38°59′41″61
PE of 1 pair (1) . . . . ±0. 31
(2) . . . . ±0. 463

Class (β). All stars AB twice observed.
All stars BB more than twice observed.

8 pairs of class (β) . . . . 38°59′41″41
PE of 1 pair (1) . . . . ±0. 38
(2) . . . . ±0. 28

Class (γ) All stars once observed . . . . 38°59′41″46
All stars AC, BC.

23 pairs of class (γ)
PE of 1 pair (1) . . . . ±0. 46
(2) . . . . ±0. 605

The probable errors (1) and (2) do not agree very well, owing to the small number of observations; but those denoted by (2) are the larger, upon the whole, owing to errors constant either for all the observations of a night, or upon a pair of stars. The three results will be thus: —

		Probable Error.	
		(1)	(2)
Class (α)	38°59′41″61	± 0″069	± 0″101
„ (β)	41.41	± 0.134	± 0.099
„ (γ)	41.46	± 0.096	± 0.126

I think no considerable uncertainty will be left if the stars of Class (β) have a weight of ¾ each, and those of Class (γ) ½ each. This will give the total probable error for weight 1, a thoroughly good pair sufficiently observed; as, —

	(1)	(2)	
From Class (α)	± 0″31	± 0″46	20 pairs.
„ „ (β)	± 0.33	± 0.24	8 „
„ „ (γ)	± 0.33	± 0.43	23 „
	Mean ± 0.32	± 0.41	51 „

and the final latitude

38°59′41″47 ± 0″067

including errors of all kinds.

The latitude is manifestly determined with all the precision necessary for an arc of the meridian; the instrument being of the largest class, the observer excellent, and the star-places the result of a careful investigation.

TABLE I.

Decl.	Log. $C^2\omega$	Log. $C\omega$	$C^2\omega$	$C\omega$	ω
0°	0.0000	0.0000	1.000	1.000	1.000
2	0.0003	0.0000	1.001	1.000	0.999
4	0.0010	0.0000	1.002	1.000	0.998
6	0.0024	0.0000	1.005	1.000	0.995
8	0.0042	0.0000	1.010	1.000	0.990
10	0.0066	9.9999	1.015	1.000	0.985
12	0.0095	9.9999	1.022	1.000	0.978
14	0.0129	9.9998	1.030	1.000	0.970
16	0.0168	9.9997	1.040	0.999	0.961
18	0.0212	9.9995	1.050	0.999	0.950
20	0.0262	9.9992	1.062	0.998	0.938
22	0.0316	9.9988	1.075	0.997	0.925
24	0.0375	9.9982	1.090	0.996	0.910
26	0.0439	9.9975	1.106	0.994	0.894
28	0.0507	9.9966	1.124	0.992	0.876
30	0.0580	9.9955	1.143	0.990	0.857
32	0.0657	9.9941	1.163	0.987	0.837
34	0.0738	9.9924	1.185	0.983	0.815
36	0.0824	9.9903	1.209	0.978	0.791
38	0.0912	9.9878	1.234	0.972	0.766
40	0.1005	9.9848	1.260	0.965	0.740
42	0.1101	9.9811	1.288	0.958	0.712
44	0.1199	9.9769	1.318	0.948	0.682
46	0.1300	9.9718	1.349	0.937	0.651
48	0.1403	9.9658	1.381	0.924	0.619
50	0.1508	9.9589	1.415	0.910	0.585
52	0.1614	9.9508	1.450	0.893	0.550
54	0.1722	9.9414	1.486	0.874	0.514
56	0.1829	9.9304	1.524	0.852	0.476
58	0.1936	9.9178	1.562	0.827	0.439
60	0.2041	9.9031	1.600	0.800	0.400
62	0.2145	9.8861	1.639	0.769	0.361
64	0.2247	9.8665	1.678	0.734	0.322
66	0.2346	9.8439	1.716	0.698	0.284
68	0.2440	9.8176	1.754	0.657	0.246
70	0.2530	9.7870	1.791	0.612	0.209
72	0.2614	9.7514	1.826	0.564	0.174
74	0.2692	9.7096	1.859	0.512	0.141
76	0.2763	9.6600	1.889	0.457	0.111
78	0.2826	9.6005	1.917	0.399	0.083
80	0.2881	9.5278	1.941	0.337	0.059
82	0.2927	9.4363	1.962	0.273	0.038
84	0.2963	9.3155	1.978	0.207	0.022
86	0.2989	9.1425	1.990	0.139	0.010
88	0.3005	8.8433	1.998	0.070	0.002
90	0.3010	———	2.000	0.000	0.000